πN sigma term and chiral-odd twist-3 distribution function $e(x)$ of the nucleon in the chiral quark soliton model

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(Received 3 December 2003; published 1 June 2004)

The isosinglet combination of the chiral-odd twist-3 distribution function $e^u(x)+e^d(x)$ of the nucleon has the outstanding properties that its first moment is proportional to the well-known πN sigma term and that it contains a δ-function singularity at $x=0$. These two features are inseparably connected in that the above sum rule would be violated if there is no such singularity in $e^u(x)+e^d(x)$. Very recently, we found that the physical origin of this δ-function singularity can be traced back to the long-range quark-quark correlation of scalar type, which signals the spontaneous chiral symmetry breaking of the QCD vacuum. The main purpose of the present paper is to give complete theoretical predictions for the chiral-odd twist-3 distribution function $e^a(x)$ of each flavor a on the basis of the chiral quark soliton model, without recourse to the derivative-expansion-type approximation. These theoretical predictions are then compared with the empirical information extracted from the CLAS data of the semi-inclusive DIS processes by assuming the Collins mechanism only. A good agreement with the CLAS data is indicative of a sizable violation of the πN sigma-term sum rule or, equivalently, the existence of a δ-function singularity in $e^u(x)+e^d(x)$.

DOI: 10.1103/PhysRevD.69.114002 PACS number(s): 12.39.Fe, 12.38.Lg, 12.39.Ki, 13.40.Em

I. INTRODUCTION

It is a widely accepted common belief now that the nonperturbative dynamics of QCD (chiral dynamics) is an indispensable element for understanding high-energy deep inelastic scattering observables. Undoubtedly, the reconfirmation of this natural fact is strongly based on to the two remarkable experimental discoveries in this field [1–3]. They are the unexpectedly small quark spin fraction of the nucleon revealed by the European Muon Collaboration (EMC) measurement [1,2] and the light-flavor sea-quark asymmetry confirmed by the New Muon Collaboration (NMC) measurement [3]. The most successful theoretical studies of parton distribution functions have been carried out within the framework of the chiral quark soliton model (CQSM) [4–15], which is an effective model of baryons maximally incorporating the spontaneous chiral symmetry breaking of the QCD vacuum. In fact, we claim that it is so far the only effective model of baryons which is able to explain the above two remarkable findings simultaneously within a single theoretical framework [16–19].

Very recently, we became aware of another novel example in which nonperturbative QCD dynamics plays an unprecedented role in the physics of parton distribution functions. It concerns the possible existence of a delta-function singularity at the Bjorken variable $x=0$ in the chiral-odd twist-3 distribution function $e(x)$ of the nucleon [20,21]. This distribution function itself, together with its first moment sum rule giving the familiar πN sigma term, has been known for a long time [22]. In spite of several interesting theoretical features, however, this distribution function has been thought of as an academic object of study, since, because of its chiral-odd nature, it does not appear in the cross section formula of inclusive deep-inelastic scattering (DIS). The situation changed drastically, however, since the CLAS Collaboration was able to get the first experimental information on this interesting quantity through measurement of the azimuthal asymmetry A_{LU} in the electroproduction of pions from deeply inelastic scattering of longitudinally polarized electrons off unpolarized protons [23–25].

Some years ago, within the framework of perturbative QCD, Burkardt and Koike noticed that the first moment sum rule (or the πN sigma-term sum rule) for $e(x)$ holds only when $e(x)$ has a δ-function-type singularity at the Bjorken variable $x=0$ [26]. Unfortunately, the physical origin of this singular term is not very clear in this perturbative analysis. Very recently, two independent proofs were given to the fact that the physical origin of this δ-function singularity can be traced back to the nonvanishing vacuum quark condensate which signals the spontaneous chiral symmetry breaking of the QCD vacuum [20,21]. An interesting question is whether we can verify experimentally the existence of this δ-function singularity in $e(x)$. Unfortunately, the point $x=0$ is experimentally inaccessible. This means that, if there really exists such a $\delta(x)$-type singularity in $e(x)$, the experimental measurement would rather confirm violation of this πN sigma-term sum rule. Nonetheless, since $e(x)$ in the region $x\neq 0$ can in principle be measured, theorists are challenged to explain its behavior.

The first theoretical study of $e(x)$ was done by using the MIT bag model [27]. (See also [28].) However, this estimate based on the bag model cannot be taken as a realistic one by the following reasons. First, its prediction for the magnitude of the πN sigma term is far from reliable. Second, more seriously, it cannot reproduce the δ-function singularity of $e(x)$. Both these features (they are not actually unrelated) are easily anticipated, since the MIT bag model is essentially a relativistic quark model with $N_c(=3)$ valence quark de-

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grees of freedom only, and the reproduction of the nonzero vacuum quark condensate is beyond the range of applicability of this model. The first realistic investigation of $e(x)$ was carried out by Efremov *et al.* on the basis of the chiral quark soliton model but within the "valence" quark only approximations [29,30]. More recently, the present authors and Schweitzer independently carried out more careful analysis of the contribution of the Dirac sea quarks on the basis of the gradient-expansion-type approximation and confirmed that the isosinglet combination of $e(x)$ certainly contains a δ-function-type singularity [20,21]. After some analysis of higher-derivative terms of the expansion, however, Schweitzer retreated to the assumption that the contribution of the Dirac sea quarks is saturated by this $\delta(x)$ term alone. As admitted by himself, however, whether this last assumption is justified or not is far from trivial [20]. To confirm it, one has to carry out an exact numerical calculation within the model without recourse to the gradient-expansion-type approximation. Furthermore, to compare the predictions of the model with the experimental data of the CLAS Collaboration, one must know $e^a(x)$ of each flavor a. To this end, only knowledge of the isoscalar combination $e^u(x)+e^d(x)$ is not enough. We need another independent combination: i.e., the isovector distribution $e^u(x)-e^d(x)$. Within the framework of the CQSM, this latter distribution survives at the next-to-leading order in $1/N_c$ expansion and it was left untouched in [20].

In view of the above-mentioned circumstances, we think it important to carry out an exact model calculation within the CQSM for both of the isoscalar and isovector combinations of the chiral-odd twist-3 distribution function $e(x)$. We also think it useful to analyze the first- and second-moment sum rule for $e^u(x)+e^d(x)$ and $e^u(x)-e^u(x)$ within the CQSM in light of the corresponding sum rule expected in the general framework of perturbative QCD. The predictions of the model for $e^u(x)$ and $e^d(x)$ (as well as the corresponding distributions for antiquarks) are then used as initial distributions given at the model energy scale around 600 MeV (or $Q^2 \simeq 0.30$ GeV^2), and they are evolved to higher Q^2 for the sake of comparison with the phenomenological information obtained by using the CLAS measurement.

The paper is organized as follows. In Sec. II, after a brief introduction of the basic idea of the CQSM, the theoretical expressions for $e^u(x)+e^d(x)$ and $e^u(x)-e^d(x)$ are given. The fundamental moment sum rules for these distributions are also discussed here in some detail. Section III is devoted to a discussion of the numerical results. Finally, in Sec. IV, we summarize what we have found.

II. $e(x)$ IN THE CHIRAL QUARK SOLITON MODEL

The chiral-odd twist-3 quark distribution $e^a(x)$ of flavor a inside a nucleon with 4-momentum P, averaged over its spin, is defined by

$$e^a(x) = P^+ \int_{-\infty}^{\infty} \frac{dz^-}{2\pi} e^{ixP^+z^-} \times \langle N|\psi_a^\dagger(0)\gamma^0\psi_a(z)|N\rangle|_{z^+=0,z_\perp=0}, \tag{1}$$

where ψ_a are quark fields. Similarly, the corresponding antiquark distribution is defined as

$$e^{\bar{a}}(x) = P^+ \int_{-\infty}^{\infty} \frac{dz^-}{2\pi} e^{ixP^+z^-} \times \langle N|\psi_a^{c\dagger}(0)\gamma^0\psi_a^c(z)|N\rangle|_{z^+=0,z_\perp=0}, \tag{2}$$

with ψ^c being the charge-conjugate field of ψ. Here we use the standard light-cone coordinates

$$z^\pm = \frac{z^0 \pm z^3}{\sqrt{2}}, \quad P^\pm = \frac{P^0 \pm P^3}{\sqrt{2}}. \tag{3}$$

The variable x denotes the Bjorken variable, $x = -q^2/(2P\cdot q)$, with q being the 4-momentum transfer to the nucleon. Taking account of the charge-conjugation property of the relevant quark bilinear operator, one can formally extend the domain of quark distribution functions from the interval $0 \leqslant x \leqslant 1$ to $-1 \leqslant x \leqslant 1$, such that

$$e^{\bar{a}}(x) = e^a(-x) \quad (0 \leqslant x \leqslant 1), \tag{4}$$

which dictates that the distribution function with negative x should be interpreted as antiquark one.

Although the above definitions of the quark and antiquark distribution functions are frame independent, it is convenient to perform the actual calculation in the nucleon rest frame. In this frame, we have $P^+ = M_N/\sqrt{2}$, and the distribution function is reduced to

$$e^a(x) = M_N \int_{-\infty}^{\infty} \frac{dz_0}{2\pi} e^{ixM_N z_0} \times \langle N|\psi_a^\dagger(0)\gamma^0\psi_a(z)|N\rangle|_{z_3=-z_0,z_\perp=0}. \tag{5}$$

Throughout the paper, we will confine ourselves to two flavor case of u and d quarks, and neglect strangeness degrees of freedom in the nucleon. Consequently, we have two independent distributions: i.e., the isosinglet distribution $e^{(T=0)}(x) \equiv e^u(x)+e^d(x)$ and the isovector one $e^{(T=1)}(x) \equiv e^u(x)-e^d(x)$. In the case of $e^{(T=0)}(x)$, we simply sum up Eq. (5) over the flavor components. On the other hand, for $e^{(T=1)}(x)$, we have to sum up the representation after inserting τ_3 matrix in Eq. (5).

For obtaining quark distribution functions, we must generally evaluate nucleon matrix elements of bilocal and bilinear quark operators containing two space-time coordinates with light-cone separation. The starting point of our theoretical analysis is the following path integral representation of the matrix elements of a bilocal and bilinear quark operator between the nucleon states with definite momentum $\boldsymbol{P}$:

$$\langle N(\boldsymbol{P})|\psi^\dagger(0)\gamma^0\psi(z)|N(\boldsymbol{P})\rangle = \frac{1}{Z}\int d^3x d^3y e^{-i\boldsymbol{P}\cdot\boldsymbol{x}} e^{i\boldsymbol{P}\cdot\boldsymbol{y}} \int \mathcal{D}U \int \mathcal{D}\psi\mathcal{D}\psi^\dagger J_N\left(\frac{T}{2},\boldsymbol{x}\right) \times \psi^\dagger(0)\gamma^0\psi(z) J_N^\dagger\left(-\frac{T}{2},\boldsymbol{y}\right) \times \exp\left[i\int d^4x \mathcal{L}\right], \tag{6}$$

where

$$\mathcal{L} = \bar{\psi}[i\partial\!\!\!/ - MU^{\gamma_5}(x)]\psi, \tag{7}$$

with

$$U^{\gamma_5}(x) = \exp[i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)/f_\pi] \tag{8}$$

being the basic Lagrangian of the CQSM with two flavors. The quantity

$$J_N(x) = \frac{1}{N_c!}\epsilon^{\alpha_1\cdots\alpha_{N_c}}\Gamma^{\{f_1\cdots f_{N_c}\}}_{JJ_3,TT_3}\psi_{\alpha_1 f_1}(x)\cdots\psi_{\alpha_{N_c}f_{N_c}}(x) \tag{9}$$

is a composite operator carrying quantum numbers JJ_3, TT_3 (spin, isospin) of the baryon, where α_i are the color indices, while $\Gamma^{\{f_1\cdots f_{N_c}\}}_{JJ_3,TT_3}$ is a symmetric matrix in spin flavor indices f_i. We start with a stationary pion field configuration of hedgehog shape:

$$\boldsymbol{\pi}(x) = f_\pi \hat{\boldsymbol{r}} F(r). \tag{10}$$

Next we carry out the path integral over $\boldsymbol{\pi}(x)$ in a saddle point approximation by taking care of two zero-energy modes: i.e., the "translational zero modes" and "rotational zero modes." Under the assumption of "slow rotation" as compared with intrinsic quark motion, the answers can be obtained in a perturbative series in Ω, which can also be regarded as a $1/N_c$ expansion. Up to first order in the collective rotational velocity Ω, the only surviving contribution to $e^{(T=0)}(x)$ arises at the $\mathcal{O}(\Omega^0)$ term of this expansion, since the $\mathcal{O}(\Omega^1)$ term vanishes identically due to the hedgehog symmetry. On the other hand, the first nonvanishing contribution to $e^{(T=1)}(x)$ arises at the $\mathcal{O}(\Omega^1)$, since the leading $\mathcal{O}(\Omega^0)$ contribution vanishes due to the hedgehog symmetry. Then, between the magnitude of the above two distributions, one may expect the following large-N_c relation:

$$|e^u(x)+e^d(x)| \sim N_c|e^u(x)-e^d(x)|. \tag{11}$$

A. Isosinglet distribution $e^{(T=0)}(x)$

The isosinglet combination of the chiral-odd twist-3 unpolarized distribution is given by

$$e^{(T=0)}(x) \equiv e^u(x)+e^d(x) = M_N\int_{-\infty}^{\infty}\frac{dz_0}{2\pi}e^{ixM_N z_0} \times \langle N|\bar{\psi}(0)\psi(z)|N\rangle|_{z_3=-z_0, z_\perp=0}. \tag{12}$$

Following the general formalism developed in [4,5,9], the isosinglet distribution in the CQSM is given in the following form:

$$e^{(T=0)}(x) = -N_c M_N \sum_{n>0}\langle n|\gamma^0\delta(xM_N - \hat{p}_3 - E_n)|n\rangle \tag{13}$$

$$= N_c M_N \sum_{n\leqslant 0}\langle n|\gamma^0\delta(xM_N - \hat{p}_3 - E_n)|n\rangle, \tag{14}$$

where $|n\rangle$ and E_n are the eigenstates and the associated eigenenergies of the static Dirac Hamiltonian

$$H = -i\boldsymbol{\alpha}\cdot\nabla + \beta M e^{i\gamma_5\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}F(r)}, \tag{15}$$

with the hedgehog background. Here, the summation $\Sigma_{n\leqslant 0}$ in Eq. (14) is meant to be taken over the valence-quark orbital (it is the lowest-energy eigenstate that emerges from the positive-energy Dirac continuum) plus all the negative-energy Dirac-sea orbitals. On the other hand, the summation $\Sigma_{n>0}$ in Eq. (13) is meant to be taken over all the positive-energy Dirac continuum excluding the discrete valence orbital. We recall that the CQSM is defined with some appropriate regularization. In fact, without regularization, $e^{(T=0)}(x)$ is quadratically divergent, and no practical meaning can be given to either of Eqs. (13) and (14). The ideal regularization scheme for our purpose is the Pauli-Villars subtraction scheme, since it preserves several fundamental conservation laws of field theory [4,5]. Furthermore, it is also expected to preserve the equivalence of the two ways of computing the quantity in question, by using Eqs. (13) and (14). In the present study, we use the double-subtraction Pauli-Villars scheme as introduced in [31], since $e^{(T=0)}(x)$ diverges like the vacuum quark condensate. In this scheme the distribution $e^{(T=0)}(x)$ is replaced with a regularized one defined as

$$e^{(T=0)}(x) \equiv e^{(T=0)}(x)^M - \sum_{i=1}^{2} c_i\left(\frac{\Lambda_i}{M}\right)e^{(T=0)}(x)^{\Lambda_i}. \tag{16}$$

Here $e(x)^{\Lambda_i}$ is obtained from $e(x)^M$ by replacing the mass parameter M by Λ_i. It was shown in [31] that, if the parameters c_1, c_2, Λ_1, and Λ_2 are chosen to satisfy the two conditions

$$1 - \sum_{i=1}^{2} c_i\left(\frac{\Lambda_i}{M}\right)^2 = 0, \tag{17}$$

$$1-\sum_{i=1}^{2} c_i\left(\frac{\Lambda_i}{M}\right)^4=0, \tag{18}$$

the quadratic as well as the logarithmic divergences in the vacuum quark condensate are completely eliminated.

Actually, we are interested in the nucleon observables measured in reference to the physical vacuum, so that $e^{(T=0)}(x)$ should be replaced by

$$e^{(T=0)}(x)\to e^{(T=0)}(x)\equiv e_U^{(T=0)}(x)-e_{U=1}^{(T=0)}(x). \tag{19}$$

Here the vacuum subtraction term $e_{U=1}^{(T=0)}(x)$ is obtained from $e_U^{(T=0)}(x)$ by setting $U=1$ or $F(r)=0$ and by excluding the sum over the discrete valence level. We point out that, as a result of the energy-momentum conservation embedded in the factor $\delta(xM_N-\hat{p}_3-E_n)$, the vacuum subtraction terms are required only for $x<0$ in the occupied form (14) and for $x>0$ in the nonoccupied form (13). This means that the vacuum subtraction terms need not be considered when $e^{(T=0)}(x)$ is evaluated in the following way—i.e., if it is evaluated by using the occupied form for $x>0$, while using the nonoccupied form for $x<0$.

Momentum sum rules of $e^{(T=0)}(x)$

The most important information of the distribution functions is generally contained in their first few moments of lowest orders. This is also the case for the distribution $e^{(T=0)}(x)$. In a recent paper, Efremov and Schweitzer reviewed some of the important sum rules for the chiral-odd twist-3 distribution functions in an enlightening way [32]. Their argument starts with the general definition of the distribution with flavor a as

$$e^a(x)=\frac{1}{2M_N}\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle N|\bar{\psi}_a(0)[0,\lambda n]\psi_a(\lambda n)|N\rangle, \tag{20}$$

where $[0,\lambda n]$ denotes the gauge link. By using an operator identity following from the QCD equation of motion, $e^a(x)$ is shown to be decomposed in a gauge-invariant way into the three pieces

$$e^a(x)=e^a_{sing}(x)+e^a_{tw3}(x)+e^a_{mass}(x). \tag{21}$$

Here $e^a_{sing}(x)$ denotes a singular term given by

$$e^a_{sing}(x)=\delta(x)\langle N|\bar{\psi}^a\psi^a|N\rangle. \tag{22}$$

On the other hand, $e^a_{tw3}(x)$ is a genuine twist-3 part of $e^a(x)$ that contains information on quark-gluon-quark correlations. Finally, $e^a_{mass}(x)$ denotes the term arising from the nonzero current quark mass. It is a somewhat peculiar function defined through its Mellin moments as [33–36]

$$\int_{-1}^{1}x^{n-1}e^a_{mass}(x)dx=\delta_{n>1}\frac{m_0}{M_N}\int_{-1}^{1}x^{n-2}f_1^a(x)dx, \tag{23}$$

with $f_1^a(x)$ being the twist-2 unpolarized distribution with flavor a. The presence of the factor $\delta_{n>1}$ here dictates that the first moment of $e^a_{mass}(x)$ vanish:

$$\int_{-1}^{1}e^a_{mass}(x)dx=0. \tag{24}$$

It is also known [33–36] that the first two basic Mellin moments of $e^a_{tw3}(x)$ vanish—i.e.,

$$\int_{-1}^{1}x^{n-1}e^a_{tw3}(x)dx=0 \quad \text{for } n=1,2. \tag{25}$$

Putting the above-mentioned properties altogether, the first-moment sum rule for the isoscalar combination of $e^a(x)$—i.e., $e^{(T=0)}(x)$—takes the form.

$$\int_{-1}^{1}e^{(T=0)}(x)dx=\frac{\Sigma_{\pi N}}{m_0}, \tag{26}$$

which is nothing but the πN sigma-term sum rule. Note that this sum rule is saturated by the first term of Eq. (21) alone. On the other hand, the second Mellin moment of $e^{(T=0)}(x)$ is given by

$$\int_{-1}^{1}xe^{(T=0)}(x)dx=\frac{m_0}{M_N}N_c, \tag{27}$$

where N_c is the number of color, which coincides with the number of quarks contained in a baryon-number-1 system—i.e., $N_c=3$. We point out that this second Mellin moment of $e^{(T=0)}$ vanishes in the chiral limit of $m_0=0$.

Next, we turn to the discussion of the moment sum rule in the CQSM. Integrating Eq. (14) over x, the first moment of $e^{(T=0)}(x)$ is given as

$$\int_{-1}^{1}e^{(T=0)}(x)dx=N_c\sum_{n\leq 0}\langle n|\gamma^0|n\rangle. \tag{28}$$

Since the right-hand side (RHS) of this equation is nothing but the scalar charge $\bar{\sigma}$ of the nucleon within the CQSM, the sigma-term sum rule immediately follows:

$$\int_{-1}^{1}e^{(T=0)}(x)dx=\bar{\sigma}=\frac{\Sigma_{\pi N}}{m_0}. \tag{29}$$

The way of this sum rule being satisfied is far more delicate in the CQSM than in the above QCD-motivated analysis. As shown by our previous study, although the model certainly predicts the $\delta(x)$-type singularity in $e^{(T=0)}(x)$, this term alone does not saturate the πN sigma-term sum rule. The model also predicts a nontrivial structure of $e^{(T=0)}(x)$ at $x\neq 0$, which may contribute to the first-moment sum rule. We shall discuss this point in more detail in the next section.

Turning to the second moment, it is easy to show from Eq. (14) that

$$\int_{-1}^{1} x e^{(T=0)}(x) dx = \frac{N_c}{M_N} \sum_{n \leqslant 0} \langle n | \gamma^0 (\hat{p}_3 + E_n) | n \rangle. \quad (30)$$

Owing to the hedgehog symmetry of the soliton, the term containing $\gamma^0 \hat{p}_3$ vanishes, and we are left with

$$\int_{-1}^{1} x e^{(T=0)}(x) dx = \frac{N_c}{M_N} \sum_{n \leqslant 0} E_n \langle n | \gamma^0 | n \rangle. \quad (31)$$

Following [20], it is convenient to rewrite the RHS of the above equation in the following manner. First, notice the identity

$$E_n \langle n | \gamma^0 | n \rangle = \frac{1}{2} \langle n | \{ \hat{H}, \gamma^0 \} | n \rangle = m_0 + M \langle n | \frac{1}{2} (U + U^\dagger) | n \rangle. \quad (32)$$

Here we have tentatively restored the current quark mass term in the model Hamiltonian H, just for the sake of explanation here only; i.e., we have used here

$$H = -i \boldsymbol{\alpha} \cdot \nabla + \beta M e^{i \gamma_5 \boldsymbol{\tau} \cdot \hat{\boldsymbol{r}} F(r)} + m_0. \quad (33)$$

Then, the second moment sum rule in the CQSM takes the form

$$\int_{-1}^{1} x e^{(T=0)}(x) dx = \frac{N_c}{M_N} (m_0 + \beta M), \quad (34)$$

with

$$\beta \equiv \sum_{n \leqslant 0} \langle n | \frac{1}{2} (U + U^\dagger) | n \rangle. \quad (35)$$

It is clear now that the RHS of this sum rule does not vanish even in the chiral limit of $m_0 = 0$, contrary to the sum rule derived from the QCD equation-of-motion method. We shall return to this point in the next section.

B. Isovector distribution $e^{(T=1)}(x)$

The isovector distribution is defined by

$$e^{(T=1)}(x) \equiv e^u(x) - e^d(x) = M_N \int_{-\infty}^{\infty} \frac{dz_0}{2\pi} e^{i x M_N z_0} \times \langle N | \overline{\psi}(0) \tau_3 \psi(z) | N \rangle \Big|_{z_3 = -z_0, z_\perp = 0}. \quad (36)$$

Within the framework of the CQSM, $e^{(T=1)}(x)$ survives only in the next-to-leading order in the collective angular velocity Ω. Following the formalism derived in [9,10], the final answer is written in the form

$$e^{(T=1)}(x) = -\langle 2T_3 \rangle_p M_N \frac{N_c}{2I} \frac{1}{3} \sum_{a=1}^{3} \sum_{m=all, n>0} \langle n | \tau_a | m \rangle \times \langle m | \tau_a \gamma^0 \left(\frac{\delta_n}{E_m - E_n} - \frac{1}{2M_N} \delta_n' \right) | n \rangle$$
$$= \langle 2T_3 \rangle_p M_N \frac{N_c}{2I} \frac{1}{3} \sum_{a=1}^{3} \sum_{m=all, n \leqslant 0} \langle n | \tau_a | m \rangle \times \langle m | \tau_a \gamma^0 \left(\frac{\delta_n}{E_m - E_n} - \frac{1}{2M_N} \delta_n' \right) | n \rangle, \quad (37)$$

with $\delta_n \equiv \delta(x M_N - E_n - \hat{p}^3)$ and $\delta_n' = \frac{\partial}{\partial x} \delta(x M_N - E_n - \hat{p}^3)$. Here I in the RHS of Eq. (37) is the moment of inertia of the soliton, given by

$$I = \frac{N_c}{6} \sum_{a=1}^{3} \sum_{m>0} \sum_{n \leqslant 0} \frac{\langle n | \tau_a | m \rangle \langle m | \tau_a | n \rangle}{E_m - E_n}. \quad (38)$$

In Eq. (37), $\langle O \rangle_p$ should be understood as an abbreviated notation of the matrix element of a collective operator O between the (spin-up) proton state—i.e.,

$$\langle O \rangle_p \equiv \int \Psi^*_{T=T_3=1/2; J=J_3=1/2}[\xi_A] O [\xi_A] \times \Psi_{T=T_3=1/2; J=J_3=1/2}[\xi_A] d\xi_A = \langle p, S_3 = 1/2 | O | p, S_3 = 1/2 \rangle. \quad (39)$$

In the present case, we have $\langle 2T_3 \rangle_p = 1$.

We immediately notice that the above expressions are not suitable for the actual numerical calculation. Here, we shall proceed as in the previous studies [9,10]. First, note that the term containing the x derivative of the δ function in Eq. (37) can be rewritten as

$$e_2(x) = -\frac{d}{dx} \frac{N_c}{4I} \frac{1}{3} \sum_a \sum_{m=all, n \leqslant 0} \langle n | \tau_a | m \rangle \langle m | \tau_a \gamma^0 \delta_n | n \rangle = \frac{1}{4 I M_N} \frac{d}{dx} e^{(T=0)}(x). \quad (40)$$

Here we have made use of the completeness of the eigenstates $|n\rangle$ of the static Dirac Hamiltonian H. [We recall that $e_2(x)$ term originates from the nonlocality in time of the operator $\overline{\psi}(0) \tau^a \psi(z)$ in Eq. (36).] It should be recognized that the x derivative of the isosinglet distribution $e^{(T=0)}(x)$ appears in the right-hand side. Since we already know that the isosinglet distribution $e^{(T=0)}(x)$ has the $\delta(x)$-type singularity connected with the nonvanishing vacuum expectation, it then follows that $e_2(x)$ has the derivative-of-$\delta(x)$-type singularity. However, it is unlikely that the net isovector distribution $e^{(T=1)}(x)$ has such a singularity, because the QCD vacuum should not violate isospin symmetry so that vacuum quark condensate of isovector type must simply vanish. This apparent discrepancy can be resolved as follows. We first divide the double sum of Eq. (37) into the sum over terms with $E_m = E_n$ and with $E_m \neq E_n$. The point is that the sum with $E_m = E_n$ in $e_1(x)$ can be rewritten in a similar form as the corresponding term in $e_2(x)$:

$$e_1(x) = M_N \frac{N_c}{2I} \frac{1}{3} \sum_a \sum_{\substack{m=all, n\leq 0 \\ (E_m \neq E_n)}} \frac{1}{E_m - E_n} \langle n | \tau_a | m \rangle \times \langle m | \tau_a \gamma^0 \delta_n | n \rangle + \frac{d}{dx} \frac{N_c}{4I} \frac{1}{3} \sum_a \sum_{\substack{m\leq 0, n\leq 0 \\ (E_m = E_n)}} \langle n | \tau_a | m \rangle \times \langle m | \tau_a \gamma^0 \delta_n | n \rangle, \quad (41)$$

$$e_2(x) = -\frac{d}{dx} \frac{N_c}{4I} \frac{1}{3} \sum_a \sum_{\substack{m=all, n\leq 0 \\ (E_m \neq E_n)}} \langle n | \tau_a | m \rangle \langle m | \tau_a \gamma^0 \delta_n | n \rangle - \frac{d}{dx} \frac{N_c}{4I} \frac{1}{3} \sum_a \sum_{\substack{m\leq 0, n\leq 0 \\ (E_m = E_n)}} \langle n | \tau_a | m \rangle \langle m | \tau_a \gamma^0 \delta_n | n \rangle. \quad (42)$$

Now, just as argued in [10,9], the $E_m = E_n$ contribution in the double sums in $e_1(x)$ and $e_2(x)$ precisely cancel each other. After regrouping the terms in such a way that this cancellation occurs at the level of analytical expressions, the $\mathcal{O}(\Omega^1)$ contribution to the distribution function $e^{(T=1)}(x) = e^u(x) - e^d(x)$ can finally be written in the following form:

$$e^{(T=1)}(x) = M_N \frac{N_c}{2I} \frac{1}{3} \sum_a \sum_{\substack{m=all, n\leq 0 \\ (E_m \neq E_n)}} \langle n | \tau_a | m \rangle \times \langle m | \tau_a \gamma^0 \left(\frac{\delta_n}{E_m - E_n} - \frac{1}{2M_N} \delta_n' \right) | n \rangle. \quad (43)$$

The fact is that, in the double sum of Eq. (42), the singularity connected with the nonzero vacuum quark condensate comes only from $E_m = E_n$ contribution: i.e., the second term of Eq. (42). As mentioned above, after the $E_m = E_n$ contributions in $e_1(x)$ and $e_2(x)$ are canceled, these singularities disappear in Eq. (43). The final theoretical formula (43) is therefore free from any singularity which contradicts the symmetries of the QCD vacuum, and it provides us with a sound basis for numerical calculation.

First moment sum rule of $e^{(T=1)}(x)$

Here we discuss the first moment sum rule of the isovector distribution. Integrating Eq. (36) over x, we obtain

$$\int_{-1}^{1} e^{(T=1)}(x) dx = \int_{-1}^{1} [e^u(x) - e^d(x)] dx = \langle N | \bar{\psi} \tau_3 \psi | N \rangle. \quad (44)$$

(Here, $\bar{\psi}\tau_3\psi$ should be taken as an abbreviated notation of $\int \bar{\psi}(\mathbf{y}) \tau_3 \psi(\mathbf{y}) d^3\mathbf{y}$, which gives the isovector scalar charge operator.) An interesting observation is that the first moment of $e^{(T=1)}(x)$ is related to the nonelectromagnetic mass difference of neutron and proton. In fact, the nonelectromagnetic neutron-proton mass difference is thought to be generated by the isospin breaking term in the QCD Hamiltonian:

$$\Delta H = \frac{m_u - m_d}{2} (\bar{\psi}_u \psi_u - \bar{\psi}_d \psi_d). \quad (45)$$

Because of the smallness of all the masses $m_u, m_d, m_d - m_u$ compared with the typical energy scale of hadron physics, we can treat ΔH as a first-order perturbation, thereby being led to the following formula for the nonelectromagnetic mass difference between neutron and proton:

$$(M_n - M_p)_{QCD} = \langle n | \Delta H | n \rangle - \langle p | \Delta H | p \rangle = (m_d - m_u) \langle p | \bar{\psi}_u \psi_u - \bar{\psi}_d \psi_d | p \rangle, \quad (46)$$

where use has been made of the isospin symmetry for the unperturbative state $|p\rangle, |n\rangle$ (i.e., the invariance under the interchanges $p \leftrightarrow n$ and $u \leftrightarrow d$). Empirically, the neutron-proton mass difference of QCD origin can be estimated from the observed mass difference by taking account of the correction due to the electromagnetic interactions:

$$(M_n - M_p)_{QCD} = (M_n - M_p)_{expt} - (M_n - M_p)_{e.m.}. \quad (47)$$

Using the values $(M_n - M_p)_{expt} \simeq 1.29$ MeV, $(M_n - M_p)_{e.m.} \simeq (-0.76 \pm 0.30)$ MeV [37], we obtain

$$(M_n - M_p)_{QCD} \simeq (2.05 \pm 0.30) \ \text{MeV}. \quad (48)$$

To extract the first moment of $e^{(T=1)}(x)$ empirically, we need to know the value of $m_d - m_u$. By using $m_d - m_u \simeq 5$ MeV, as an order-of-magnitude estimate, we obtain

$$\int_{-1}^{1} e^{(T=1)}(x) dx = \frac{(M_n - M_p)_{QCD}}{m_d - m_u} \simeq 0.41 \pm 0.06. \quad (49)$$

On the other hand, the theoretical expression for the first moment of $e^{(T=1)}(x)$ is obtained from Eq. (43) as

$$\int_{-1}^{1} e^{(T=1)}(x) dx = \frac{N_c}{2I} \frac{1}{3} \sum_a \sum_{n\leq 0} \sum_{m>0} \frac{\langle n | \tau_a | m \rangle \langle m | \tau_a \gamma^0 | n \rangle}{E_m - E_n}. \quad (50)$$

Here we have used the fact that, since the contribution $e_2(x)$ is a total derivative, it does not contribute to the integral of Eq. (50). After integration over x, the double sum over levels in Eq. (50) is naturally restricted to include only transitions from occupied to nonoccupied states. This is reasonable, since the operator appearing on the RHS of Eq. (50) is a local operator, and transitions from occupied to occupied states would violate the Pauli principle. Within the framework of the CQSM, we can evaluate the RHS of Eq. (44)—i.e., the isovector scalar charge of the nucleon $\langle N | \bar{\psi} \tau^3 \psi | N \rangle$—directly without passing through the distribution function. Since the resultant expression of $\langle N | \bar{\psi} \tau^3 \psi | N \rangle$ precisely coincides with the RHS of Eq. (50), we conclude that the first moment sum rule of $e^{(T=1)}(x)$ is properly satisfied within the model.

III. NUMERICAL RESULTS AND DISCUSSION

The numerical method used for evaluating $e(x)$ in this paper is essentially the same as the one used for computing the twist-2 distributions $q(x), \Delta q(x), \delta q(x)$ [8,9]. The eigenenergies and eigenvectors of the static Dirac Hamiltonian H with the hedgehog background are obtained by diagonalizing it with the so-called Kahana-Ripka plane-wave basis [38]. Following them, the plane-wave states, introduced as a set of eigenstates of the free Hamiltonian $H_0 = -i\boldsymbol{\alpha}\cdot\nabla + \beta M$, are discretized by imposing an appropriate boundary condition for the radial wave functions at the radius D chosen to be sufficiently larger than the soliton size. The basis is made finite by retaining only those states with the momentum k satisfying the condition $k < k_{max}$. As a result of using this discretized momentum basis, the resultant distribution becomes a discontinuous function of x, due to the factor $\delta(xM_n - E_n - \hat{p}_3)$. In order to get a continuous function with a discretized basis, we introduce a smeared distribution function in the variable x as [5]

$$e_\gamma(x) \equiv \frac{1}{\gamma\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-(x-x')^2/\gamma^2} e(x') dx', \tag{51}$$

with a small but finite value of γ ($\gamma \ll 1$). The smeared distribution is expected to be continuous when the separation between the discretized momenta is much smaller than the smearing width γ. Since the physical distribution corresponds to the limit $\gamma \to 0$, this forces us to employ a very large box size D to get a continuous distribution function.

This procedure works very well at least for the standard distributions investigated so far. However, in the numerical calculation of $e^{(T=0)}(x)$, we have a new problem which we have not encountered before. Our expectation is that, if a $\delta(x)$-type singularity really exists in $e^{(T=0)}(x)$, the corresponding smeared distribution would have a Gaussian peak centered around $x=0$ with width γ. The problem here is that the distribution function in question may also have a piece that is nonsingular for all values of x. One might think that the contribution of the singular part can be disentangled from the total contribution by using the "unsmearing method" described in [5]. This is not feasible, however, for the following reasons. First, although the smearing procedure defined by Eq. (51) preserves the integral value of the distribution, we have no *ad hoc* way of knowing the overall coefficient of the $\delta(x)$ term of the distribution. Second, the small-x behavior of the nonsingular part of the distribution would be hard to know, because it is buried in the very large contribution of the smeared δ-function singularity. This point will be discussed in more detail in the following subsection.

A. Isosinglet distribution $e^{(T=0)}(x)$

In the numerical calculation, we fix the pion weak decay constant f_π in Eq. (10) to its physical value—i.e., $f_\pi = 93$ MeV—so that only one parameter of the model is the dynamical quark mass M, which plays the role of the coupling constant between the pion and effective quark fields. Through the present analysis, we use the value of $M = 375$ MeV, which is favored from the phenomenology of nucleon low-energy observables. With $M = 375$ MeV, we have $\Lambda_1 \simeq 627$ MeV and $\Lambda_2 \simeq 1589$ MeV from the conditions (17), (18). The static soliton energy obtained with these parameters is about 1018 MeV. We point out that, although the soliton mass emerges about 8% larger than the observed nucleon mass M_N, the consistency with the energy-momentum sum rule of the unpolarized distribution functions enforces us to use this value for M_N in the following evaluation of the distribution functions.

We start with showing the numerical equivalence of the final answers based on the nonoccupied representation and the occupied one. The problem here is the dependence on the cutoff momentum k_{max}, which is introduced to make finite the discretized Kahana-Ripka basis set. Since the distribution $e^{(T=0)}(x)$ is ultraviolet finite after the introduction of the double-subtraction Pauli-Villars regularization, one might expect that the answers would be stable as far as one takes k_{max} much larger than the second Pauli-Villars cutoff mass $\Lambda_2 \simeq 1.6$ GeV. This is not the case, however. As clarified in [21], the δ-function-type singularity in $e^{(T=0)}(x)$ is generated by the contribution of the infinitely deep Dirac-sea levels, which are naturally contained in either of the three terms: i.e., the main term and the two Pauli-Villars subtraction terms. This implies that the singularity, which will appear in the smeared distribution as a Gaussian peak around $x=0$ with width γ, would be reproduced only in the ideal limit of $k_{max} \to \infty$. To achieve this ideal limit, we therefore use an extrapolation method explained below. For this extrapolation to be done smoothly, we first introduce an energy cutoff into the level sums (13) and (14) of the form

$$[e^u(x) + e^d(x)]^R_{nonoccupied} = -N_c M_N \sum_{n>0} \langle n | \gamma^0 \delta(xM_N - \hat{p}_3 - E_n) | n \rangle R(E_n), \tag{52}$$

$$[e^u(x) + e^d(x)]^R_{occupied} = N_c M_N \sum_{n \le 0} \langle n | \gamma^0 \delta(xM_N - \hat{p}_3 - E_n) | n \rangle R(E_n). \tag{53}$$

Here $R(E_n)$ is a smooth regulator function with an energy cutoff $E_{max} = \sqrt{k_{max}^2 + M^2}$. For this regulator function, we employ here a Gauusian function

$$R(E_n) = \exp[-(E_n/E_{max})^2], \tag{54}$$

following Diakonov *et al.* [5]. We first compute the level sums (52) and (53) for several values of k_{max}, in the case of masses M, Λ_1, and Λ_2, respectively, and then perform the Pauli-Villars subtraction, and finally remove the energy cutoff by the numerical extrapolation to infinity pointwise in x. In the present study, we use five data (corresponding to $k_{max}/M = 12$, 16, 20, 24, and 28) and perform a least-squares fit of these data by using a fourth-order function of $1/k_{max}$.

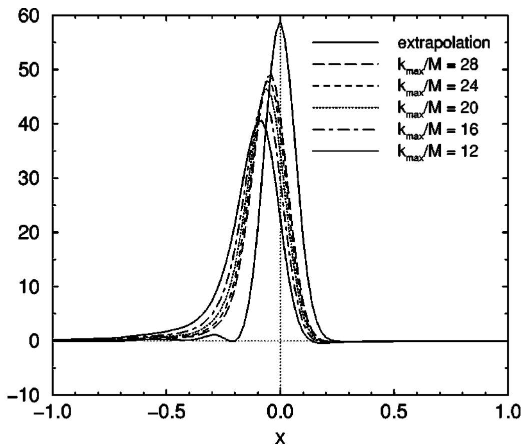

FIG. 1. The k_{max} dependence of the Dirac-sea contribution to $e^{(T=0)}(x)$ based on the occupied representation. The solid curve represents the extrapolated result.

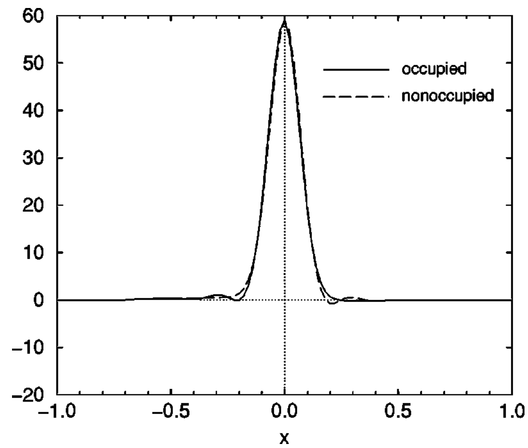

FIG. 2. Comparison of the Dirac-sea contributions to $e^{(T=0)}(x)$ based on the occupied (solid curve) and nonoccupied (dashed curve) representations.

Now we are ready to show in Fig. 1 the k_{max} dependence of the Dirac-sea contributions based on the occupied representation for all values of x. Here we use a value of $\gamma = 0.1$. This figure shows that the peak positions of the Gaussian-like function obtained with the finite cutoff energy deviate to the negative-x region from the origin $x=0$. This deviation of the peak position in the smeared distribution may be understood as follows. First, when one uses the occupied representation, the vacuum substraction as represented by Eq. (19) is necessary only for the region $x<0$, while it is not necessary for $x>0$, since the vacuum term identically vanishes for $x>0$ due to the restriction of the factor $\delta(xM_N - E_n - \hat{p}_3)$. Second, we recall the fact that the singular term of $e^{(T=0)}(x)$ emerges as a delicate cancellation of two large numbers or infinities—i.e., the difference between the main contribution with hedgehog background and the vacuum subtraction term obtained with $U=1$. These two facts indicate that the use of the occupied form with some finite value of γ can reproduce the redistribution of the delta-function strength at $x=0$ in the $x<0$ region only, but it cannot do it properly in the $x>0$ region, as far as the finite energy cutoff is used. This is the reason why the Gaussian-like peak of the smeared distribution is shifted to the negative-x region. One can, however, confirm the behavior that the position of the Gaussian peak approaches $x=0$ as the energy cutoff is increased. And, finally, with the extrapolation method, we obtain a reasonable result which shows that the peak of the smeared distribution is positioned just at $x=0$. [In the above analysis, we fix the box size to be $DM = 20$. As a matter of course, to get physically acceptable answers, we must also investigate the dependence of the answers on the box size D. We found that, above $DM=20$, the change of the small-x behavior of $e^{(T=0)}(x)$ as illustrated in Fig. 1 is almost due to the increase of k_{max}, and the answer is stable against the further increase of DM above 20.]

After carrying out a similar analysis, this time, with use of the nonoccupied representation, we can now compare the final numerical results for the Dirac-sea contribution obtained with the two alternative representations. Figure 2 shows this comparison. A reasonable agreement between the two ways of evaluating $e^{(T=0)}(x)$ confirms the equivalence of the two representations. At the same time, the analysis above establishes the existence of the δ-function singularity in $e^u(x) + e^d(x)$ on numerical grounds. Some difference between the two curves at the positive- and negative-x tails of the Gaussian-like distributions would be a spurious one generated by the numerical extrapolation method. The contributions based on the occupied representation for $x<0$ and the nonoccupied representation for $x>0$ can be obtained after cancellation of two large numbers: i.e., the main contribution and the corresponding vacuum subtraction term. On the other hand, if one uses the occupied representation for $x>0$ and the nonoccupied representation for $x<0$, one is free from the spurious contribution due to the cancellation, so that the extrapolated curves at these tail regions have reasonable smooth behavior.

Although we were able to confirm the existence of a $\delta(x)$-type singularity in the numerical analysis of $e^{(T=0)}(x)$, we cannot exclude the possibility that the $e^{(T=0)}(x)$ may also contain a regular term which is smooth in all the range of x. Is it possible to disentangle such a nonsingular term of $e^{(T=0)}(x)$ from the total contribution containing the singular one? One should recognize that it is not so easy for the following reasons. First, the deconvolution method as proposed by Diakonov *et al.* does not work because of the very delicate nature of the singularity [5]. Second, we have no *ad hoc* way to know the coefficient of $\delta(x)$ term in the original unsmeared distribution. Nevertheless, we found that the following trick works for obtaining the nonsingular distribution excluding the $\delta(x)$ term. That is, as repeatedly emphasized, by using the nonoccupied expression for $x<0$ and the occupied one for $x>0$, we can avoid the vacuum subtraction. Interestingly, this also works to remove the singular contribution in the bare distribution, and the corresponding smeared distribution would not contain the Gaussian peak corresponding to the $\delta(x)$-type singularity. [One should remember the fact that the vacuum term plays an indispensable role in reproducing the δ-function singularity in $e^{(T=0)}(x)$.]

Figure 3 shows the k_{max} dependence of the Dirac-sea contributions based on the occupied representation for $x>0$ and

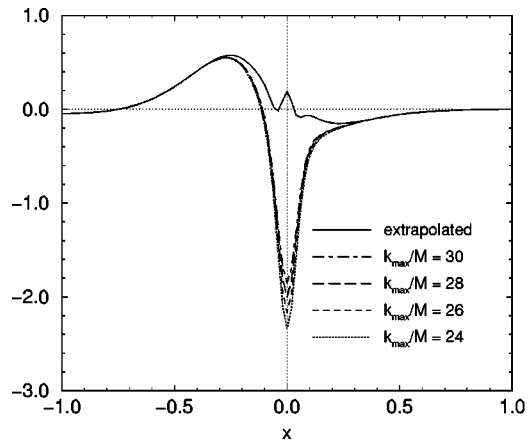

FIG. 3. The k_{max} dependence of the Dirac-sea contributions to $e^{(T=0)}(x)$ based on the occupied representation for $x>0$ and the nonoccupied representation for $x<0$. The solid curve represents the extrapolated result.

the nonoccupied representation for $x<0$. One finds that the large and positive Gaussian peak, the reminiscence of the δ-function singularity in the bare distribution, does not appear anymore. One can also see that the negative large contributions of the Dirac sea in the small x region tend to decrease as the cutoff momentum k_{max} increases. We again remove the energy cutoff by numerical extrapolation to infinity pointwise in x. We observe some difference from the previous case, however. Owing to the feature that the δ-function singularity is already excluded in the present way of calculation, the k_{max} dependence is well reproduced by the linear function of $1/k_{max}$ as illustrated in Fig. 4. After this extrapolation procedure, the result shows a smooth behavior in the whole region of x except the region $|x|<0.06$ in which the answer is thought to contain some numerical instability generated by the extrapolation method. Neglecting the data in the $|x|<0.06$ region, we make this extrapolated result smooth. After deconvoluting the smeared distribution with

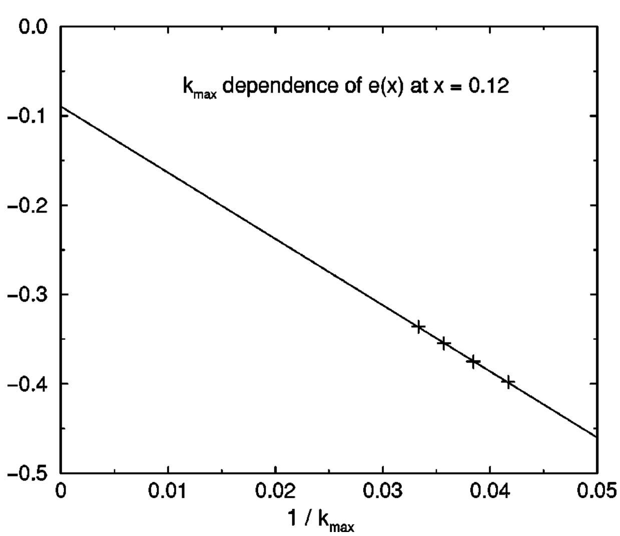

FIG. 4. The k_{max} dependence of $e_{sea}^{(T=0)}(x)$ at $x=0.12$ and its linear extrapolation to $k_{max}\to\infty$.

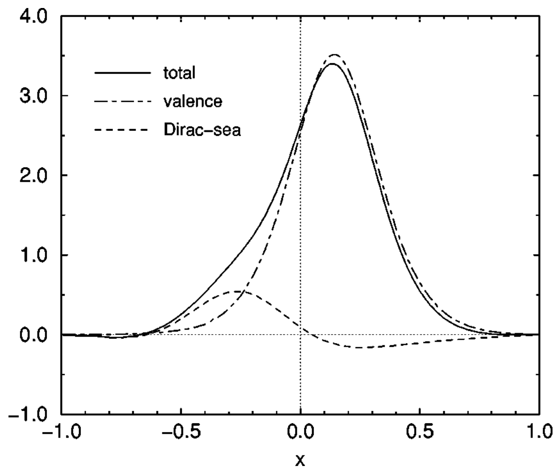

FIG. 5. The final theoretical predictions of the CQSM for $e^{(T=0)}(x)$. The dot-dashed curve represents the contribution of N_c valence level quarks, the dashed curve the nonsingular part of the Dirac-sea contributions, and the solid curve their sum. The δ-function singularity at $x=0$ in the Dirac-sea-quark part is not shown in this figure.

use of the Fourier and its inverse transforms, we obtain the final prediction for the distribution $e^{(T=0)}(x)$ within the framework of the CQSM, the normalization point of which may be interpreted as about 600 MeV.

Summarizing our analysis up to this point, the isosinglet part of the chiral-odd twist-3 distribution is given as a sum of the valence-quark and Dirac-sea-quark contributions,

$$e^{(T=0)}(x)=e_{val}^{(T=0)}(x)+e_{sea}^{(T=0)}(x), \tag{55}$$

where the Dirac-sea contribution consists of the singular term and the nonsingular (regular) term as

$$e_{sea}^{(T=0)}(x)=C\,\delta(x)+e_{reg}^{(T=0)}(x). \tag{56}$$

Shown in Fig. 5 are the final theoretical predictions for $e^{(T=0)}(x)$ obtained in the above-explained way. The dashed curve here represents the contribution of N_c valence level quarks, while the dotted curve does the regular part of Dirac-sea contribution. The sum of these two contributions is shown by the solid curve. [We recall that the $\delta(x)$-type singular term is not shown in this figure.] One can convince oneself that the regular part of the Dirac-sea contribution shows a nontrivial structure in the $x\neq 0$ region.

After performing the numerical integration of the above distributions over x, one can obtain the contributions of the valence quark term and the regular part of the Dirac-sea term to the first-moment sum rule:

$$\int_{-1}^{1} e_{val}^{(T=0)}(x)dx \simeq 1.7, \tag{57}$$

$$\int_{-1}^{1} e_{reg}^{(T=0)}(x)dx \simeq 0.18. \tag{58}$$

Note that the regular part of $e_{sea}^{(T=0)}(x)$ gives a small but nonzero contribution to the sum rule. To determine the coefficient of the singular term in Eq. (56), we use the first-moment sum rule (28) or (29) for $e^{(T=0)}(x)$, which was already shown to hold within the framework of the CQSM. We first recall that the RHS of the sum rule (28) or (29) is the nucleon scalar charge defined by

$$\bar{\sigma}=\langle N|\bar{\psi}_u\psi_u+\bar{\psi}_d\psi_d|N\rangle. \tag{59}$$

The point is that this low-energy observable can be calculated within the CQSM, without asking for the distribution function $e^{(T=0)}(x)$. It is given as

$$\bar{\sigma}=\bar{\sigma}_{val}+\bar{\sigma}_{sea}, \tag{60}$$

with

$$\bar{\sigma}_{val}=N_c\langle 0|\gamma^0|0\rangle, \tag{61}$$

$$\bar{\sigma}_{sea}=N_c\sum_{n<0}\langle n|\gamma^0|n\rangle. \tag{62}$$

Numerically, we find that

$$\bar{\sigma}_{val}\simeq 1.7, \quad \bar{\sigma}_{sea}\simeq 10.1, \tag{63}$$

so that

$$\bar{\sigma}=\bar{\sigma}_{val}+\bar{\sigma}_{sea}\simeq 11.8. \tag{64}$$

Then, by admitting the validity of the first-moment sum rule, one can extract the coefficient of the $\delta(x)$ term as follows:

$$C=\bar{\sigma}_{sea}-\int_{-1}^{1}e_{reg}^{(T=0)}(x)dx\simeq 9.92. \tag{65}$$

Our procedure for obtaining the coefficient C is different from that of Schweitzer [20]. After some consideration based on the gradient expansion analysis, he assumed that the Dirac-sea contribution to $e^{(T=0)}(x)$ is saturated by the $\delta(x)$ term with the coefficient $\Sigma_{\pi N}/m_0$, and simply neglected the possible existence of the nonsingular contribution. In his treatment, then, the nontrivial shape of $e^{(T=0)}(x)$ at $x\neq 0$ solely comes from the contribution of N_c valence level quarks. Thus, the total distribution consists of these two terms as

$$e^{(T=0)}(x)=\frac{\Sigma_{\pi N}}{m_0}\delta(x)+e_{val}(x). \tag{66}$$

(Here for simplicity, we ignore the term proportional to the product of m_0 and the unpolarized distribution function.) In our opinion, this procedure has a danger of double counting. Within the framework of the CQSM, the total πN sigma term divided by the current quark mass m_0 is nothing but the total scalar charge $\bar{\sigma}$ of the nucleon, which is made up of the two terms $\bar{\sigma}_{val}$ and $\bar{\sigma}_{sea}$. The x integral of Eq. (66) would then lead to

$$\int_{-1}^{1}e^{(T=0)}(x)dx=(\bar{\sigma}_{val}+\bar{\sigma}_{sea})+\bar{\sigma}_{val}, \tag{67}$$

which is obviously double counting the valence quark contribution to the first-moment sum rule. From a practical viewpoint, this double counting is not so serious, since the $\bar{\sigma}_{val}$ term turns out to be an order of magnitude smaller than $\bar{\sigma}_{sea}$. This dominance of the Dirac-sea contribution to the nucleon scalar charge is one of the distinguishing features of the CQSM. One can say that it is connected with the unique feature of this model, which is able to describe simultaneously a localized baryonic excitation together with the nontrivial QCD vacuum structure with nonzero quark condensate (or nonzero scalar quark density). In any case, we emphasize that the CQSM predicts a fairly large scalar charge for the nucleon: i.e., $\bar{\sigma}\simeq 11.8$. Using the current quark mass of $m_0\simeq 5$ MeV as an estimate, this gives

$$\Sigma_{\pi N}\equiv m_0\bar{\sigma}\simeq 60 \ \text{MeV}, \tag{68}$$

which seems to favor relatively large values obtained from a recent analysis of the pion-nucleon scattering amplitude [39–43].

Next we turn to the discussion of the second-moment sum rule. We first point out that the $\delta(x)$ term in $e^{(T=0)}(x)$ does not contribute to the second moment. In the CQSM, then, the second moment of $e^{(T=0)}(x)$ receives contributions from two terms in the distribution: i.e., the valence-quark term $e_{val}^{(T=0)}(x)$ and the regular part of the vacuum polarization term $e_{reg}^{(T=0)}(x)$. After performing the numerical integration, we find that

$$\int_{-1}^{1}xe_{val}^{(T=0)}(x)dx\simeq 0.23, \tag{69}$$

$$\int_{-1}^{1}xe_{sea}^{(T=0)}(x)dx=\int_{-1}^{1}xe_{reg}^{(T=0)}(x)dx\simeq -0.05. \tag{70}$$

The total second moment is therefore given by

$$\int_{-1}^{1}xe^{(T=0)}(x)dx\simeq 0.23-0.05\simeq 0.18. \tag{71}$$

We recall that, within the CQSM, there is another independent method for evaluating the second moment. Since we are working in the chiral limit, we rewrite Eq. (34), by setting $m_0=0$, as

$$\int_{-1}^{1}xe^{(T=0)}(x)dx=N_c\frac{M}{M_N}\beta \tag{72}$$

or

$$\int_{-1}^{1}xe_{val}^{(T=0)}(x)dx=N_c\frac{M}{M_N}\beta_{val}, \tag{73}$$

$$\int_{-1}^{1}xe_{sea}^{(T=0)}(x)dx=N_c\frac{M}{M_N}\beta_{sea}, \tag{74}$$

with

$$\beta_{val}=\langle 0|\frac{1}{2}(U+U^{\dagger})|0\rangle, \tag{75}$$

$$\beta_{sea}=\sum_{n<0}\langle n|\frac{1}{2}(U+U^{\dagger})|n\rangle. \tag{76}$$

These quantities β_{val} and β_{sea} can be calculated directly within the model, without invoking the corresponding distribution functions. Numerically, we find that

$$N_c\frac{M}{M_N}\beta_{val}\simeq 0.23, \tag{77}$$

$$N_c\frac{M}{M_N}\beta_{sea}\simeq -0.06. \tag{78}$$

These two numbers are consistent with the corresponding numbers in Eqs. (69) and (70), obtained through the distribution functions. A small discrepancy between the numbers in Eqs. (70) and (78) may be interpreted as giving a measure of numerical errors introduced by the very delicate interpolation method for obtaining the vacuum polarization term of $e^{(T=0)}(x)$. At any rate, we find that the CQSM predicts a relatively small but nonzero value for the second moment of $e^{(T=0)}(x)$. Since we are working in the chiral limit ($m_0=0$), this appears to contradict the corresponding sum rule (27) derived on the basis of the QCD equations of motion, which states that the second moment of $e^{(T=0)}(x)$ vanishes in the chiral limit. Does this discrepancy simply mean the limitation of the CQSM as an effective theory of QCD? In our opinion, this is not necessarily the case by the following reasons. First of all, we point out that moment sum rules containing quark masses are somewhat delicate, since the masses are generally dependent on the renormalization scale. Second, if the QCD vacuum breaks the chiral symmetry spontaneously as is generally believed, a quark acquires a dynamical mass of several hundred MeV, which means that massless quarks are nowhere. Naturally, the situation is not so simple because of the color confinement. For instance, according to the picture of the MIT bag model, which realizes quark confinement by hand, at least the vacuum inside the bag is perturbative and the quarks inside it remains massless. According to Shuryak [44], the bag model is based on the idea that the hadron is a piece of a qualitatively different (or "perturbative") phase of the QCD vacuum. The physical picture of the CQSM for the baryon and the QCD vacuum is fairly different from that of the bag model. According to the words of Shuryak again, the chiral models (including the CQSM) assume that the vacuum is only slightly modified inside the hadron: the relative orientation of the right- and left-handed quark fields is somewhat different. This last statement denotes the fact that, in the basic Lagrangian of the CQSM, the dynamical quark mass parameter M appears as a product with the chiral field $U^{\gamma_5}(x)$, which is space-time dependent. It is also the cause of the fact that the product of M and β enters the RHS of the second-moment sum rule (34). This supports Schweitzer's viewpoint [20] that the quantity βM can be interpreted as an effective mass of quarks bound in the soliton background at least in the second-moment sum rule of $e^{(T=0)}(x)$. Numerically, we have

$$\beta M \sim 51 \text{ MeV}. \tag{79}$$

This value is smaller than the one obtained in [20], since the contribution of the Dirac-sea quarks neglected in [20] works to reduce the value of β.

In any case, the nonzero value of the second moment of $e^{(T=0)}(x)$ is not contradictory at least within the framework of the CQSM in which massless quarks are nowhere at the model energy scale of about 600 MeV. However, we anticipate that the dynamical quark mass M is generally a scale-dependent quantity which approaches zero in the high-energy limit. The naive QCD sum rule for the second moment of $e^{(T=0)}(x)$ would be recovered in this limit. To verify the validity of this idea, what is crucial is experimental determination of the second-moment sum rule at the relatively low-energy scale close to the above-mentioned model energy scale. This may be in principle possible by inversely evoluting high-energy data to low-energy scale.

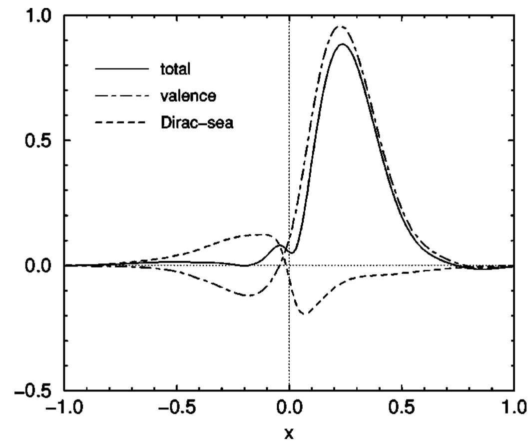

FIG. 6. The theoretical predictions of the CQSM for $e^{(T=1)}(x)$. The dot-dashed curve stands for the contribution of N_c valence level quarks, the dashed curve the contribution of the Dirac-sea quarks, while the solid curves represents their sum.

B. Isovector distribution $e^{(T=1)}(x)$

In the case of the isovector distribution $e^{(T=1)}(x)$, no ultraviolet regularization is needed because its first moment (50) is related to the imaginary part of the Euclidian effective meson action in the background soliton field [45] and it is ultraviolet finite. We have checked that the energy level sum (43) is stable enough against an increase of the cutoff momentum k_{max}, above $12M$. The final result for the isovector distribution $e^{(T=1)}(x)$ is shown in Fig. 6.

The dashed curve represents the contribution of the N_c valence level quarks, and the dot-dashed curve represents the contribution of the Dirac-sea quarks, while the solid curve represents their sum. In contrast to the isosinglet distribution, the Dirac-sea contribution has no singularity at $x=0$ and it is a smooth function in the whole region of x. The total contribution is given by the solid curve.

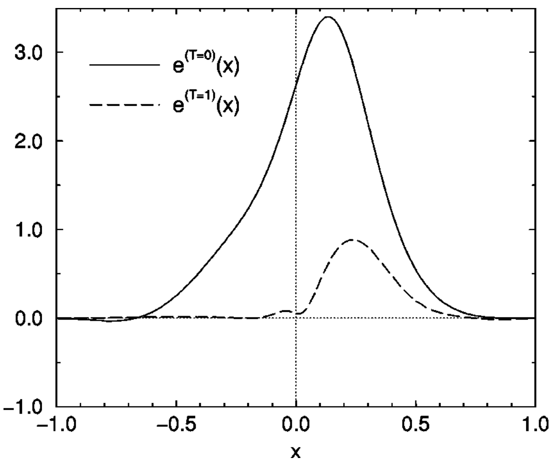

FIG. 7. The comparison of the theoretical predictions for $e^{(T=0)}(x)$ and $e^{(T=1)}(x)$ at the model energy scale.

The first moment or the x integral of this total contribution gives the value

$$\int_{-1}^{1} e^{(T=1)}(x)dx \simeq 0.28, \tag{80}$$

which is order of magnitude consistent with the estimate obtained from the analysis of the nonelectromagnetic proton-neutron mass difference. Shown in Fig. 7 are a comparison of our final theoretical predictions for $e^{(T=0)}(x)$ and $e^{(T=1)}(x)$. One confirms that the magnitude of $e^{(T=1)}(x)$ is much smaller than that of $e^{(T=0)}(x)$ in conformity with the large-N_c relation (11). Combining these two distributions, we can now give final theoretical predictions for the chiral-odd twist-3 distribution function $e^a(x)$ of each flavor a. Shown in Fig. 8(a) are the distributions for the u quark and $\bar{u}$ quark, while Fig. 8(b) gives the distributions for the d quark and $\bar{d}$ quark.

C. Comparison with empirical information from the CLAS measurements

Here we make a very preliminary comparison of our theoretical predictions for $e(x)$ with empirical information extracted from high-energy semi-inclusive scatterings. Because of its chiral-odd nature, the distribution function $e(x)$ does not appear in inclusive DIS cross sections. To extract any information for it, we must therefore carry out more specific semi-inclusive-type scattering experiments. Very recently, such an experiment has in fact been done by the CLAS Collaboration [23]. They measured the azimuthal asymmetry A_{LU} in electroproduction of pions from deeply inelastic scatterings of longitudinally polarized electrons off unpolarized protons.

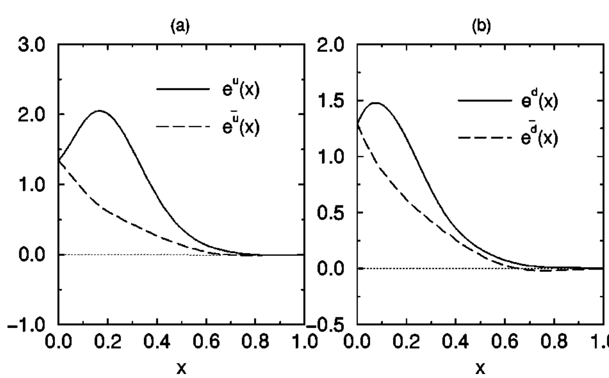

FIG. 8. The theoretical predictions for $e^u(x)$, $e^d(x)$, $e^{\bar{u}}(x)$, and $e^{\bar{d}}(x)$ at the model energy scale.

The first theoretical analysis of the CLAS data was carried out by Efremov *et al.* [29,30]. Their analysis assumes that the beam single-spin asymmetry measured by the CLAS group is dominantly generated by the so-called Collins mechanism [46]. Under this assumption together with a particular parametrization for the Collins fragmentation function, they were able to extract the first information on the chiral-odd twist-3 distribution function $e(x)$. Recently, this analysis was criticized by Yuan [47]. According to him, there may be another mechanism which competes with the Collins mechanism [48,49]. It is the leading-order transverse-momentum-dependent parton distribution $h_1^{\perp}(x,k_{\perp})$ convoluted with chiral-odd fragmentation function $\hat{e}(z)$. After all, the fact is that we still have poor knowledge about the mechanism that generates the beam single-spin asymmetry in semi-inclusive deep-inelastic scatterings. We must understand the mechanism of parton fragmentation processes into hadrons, especially the physics of time-reversal-odd fragmentation functions [46,50]. We must also clarify the dynamics of transverse-momentum-dependent parton distribution functions in combination with the physics of chiral-odd fragmentation functions [48–50]. A truly reliable extraction of the chiral-odd twist-3 distribution function $e(x)$, which is of our primary concern here, can be achieved only after a more complete understanding of the above-mentioned mechanisms of semi-inclusive DIS processes.

Keeping this fact in mind, we shall proceed here by assuming dominance of the Collins mechanism. Under this assumption, the asymmetry measured by the CLAS experiment is interpreted to be proportional to

$$A_{LU}^{\sin\phi} \sim -\frac{4\pi\alpha^2 s}{Q^4}\lambda_e 2y\sqrt{1-y}\sum_a e_a^2 x^2 e^a(x) H_1^{\perp a}(z), \tag{81}$$

with $y=(P\cdot q)/(P\cdot l), z=(P\cdot p_h)/(P\cdot q)$ and s is the invariant mass squared of the photon-hadron system in the notation of [29]. λ_e denotes the beam helicity. The chiral-and T-odd twist-2 "Collins" fragmentation function $H_1^{\perp a}(z)$ gives the probability of a spinless or unpolarized hadron to be created from a transversely polarized scattered quark. Using information on $H_1^{\perp a}(z)$ from HERMES data [24,25], one can then get direct information on the distribution function $e(x)$ [29,30]. In the CLAS experiment, the azimuthal asymmetries $A_{LU}^{\sin\phi}$ for the process $\vec{e}p \to e'\pi^+ X$ were measured at $Q^2 \sim 1.5\ \mathrm{GeV}^2$. Under the dominant-flavor-only approximation for the fragmentation functions, the semi-inclusive π^+ production measures the following combination of distributions:

$$e^u(x) + \frac{1}{4}e^{\bar{d}}(x). \tag{82}$$

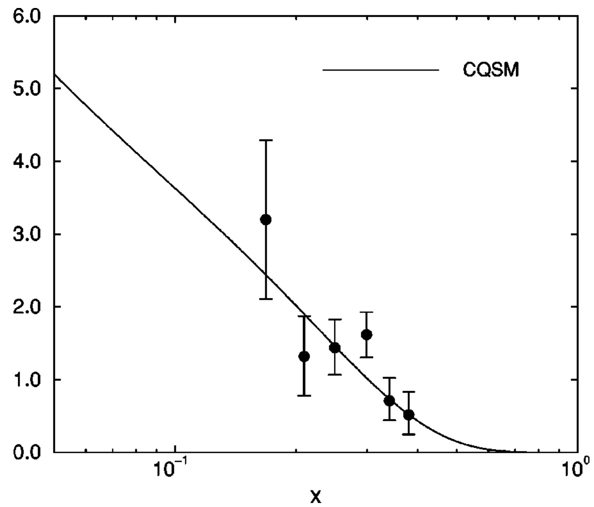

FIG. 9. The theoretical prediction for $e(x)=e^u(x)+\frac{1}{4}e^{\bar{d}}(x)$ in comparison with the corresponding empirical information extracted from the CLAS data at $\langle Q^2\rangle=1.5$ GeV^2 under the assumption of Collins mechanism dominance.

In Fig. 9, we make a comparison between the predictions of the CQSM for the above combinations of the distributions and the corresponding empirical information extracted from the CLAS data by Efremov *et al.* [29,30] under the assumption of Collins mechanism dominance. The theoretical distribution here corresponds to an energy scale of $Q^2=1.5$ GeV^2. The scale dependence of the distribution is taken into account by solving the leading-order DGLAP-type equation obtained in the large-N_c limit [35]. (The starting energy scale of this evolution is taken to be $Q^2_{ini}\simeq 0.30$ GeV^2.) The distribution $e^u(x)+\frac{1}{4}e^{\bar{d}}(x)$ extracted from the CLAS data contains large errors mainly due to the large uncertainties of $H_1^{\perp}(z)$ from the HERMES data [24,25]. Still, it was emphasized in [29,30] that the extracted distribution is definitely larger than the "twist-3 bound" and about 2 times smaller than the corresponding unpolarized distribution $f_1^a(x)$ at the same energy scale. One sees that our theoretical prediction for $e^u(x)+\frac{1}{4}e^{\bar{d}}(x)$ is in fairly good agreement with the extracted behavior from the CLAS data. The relatively small magnitude of the extracted $e(x)$ indicates that there must be a significant contribution to the πN sigma-term sum rule from the small-x region. Whether this is due to the indicated δ-function singularity in $e(x)$ or it is due to yet-unresolved Regge behavior in the small-x region is difficult to judge at the present stage of study. It is highly desirable to extend the region of measurements to a smaller-x region. This is important, because unambiguous establishment of the violation of the πN sigma-term sum rule would indirectly prove the existence of a novel δ-function singularity in the distribution function $e(x)$ of the nucleon, which in turn may be interpreted as a manifestation of the nontrivial structure of QCD vacuum in an observable of a localized QCD excitation: i.e., the nucleon.

Finally, we want to make some comments on the prediction for $e(x)$ based on the MIT bag model. As mentioned in [30], the bag model prediction of [28] evolved to the comparable energy scale of $Q^2=1$ GeV^2 is in qualitative agreement with the extracted $e(x)$ from the CLAS data in [30]. In our opinion, this agreement should be taken as fortuitous for the following reason. First, as already pointed out, the isosinglet scalar charge of the nucleon predicted by the MIT bag model is only about 15% of the value expected from the phenomenological knowledge of the πN sigma term. The fact is that the nucleon isoscalar charge is a quantity of order 1 (or order N_c, more precisely) in the MIT bag model or in any other model which contains three valence-quark degrees of freedom only. The situation is totally different in the CQSM. Although the contribution of the N_c valance level quarks is of the same order as that of the MIT bag model, the vacuum polarization effect or the contribution of the Dirac-sea quarks gives a nearly 7-times-larger contribution as compared with that of the valence quarks, thereby reproducing the correct magnitude of the nucleon scalar charge or the πN sigma term. Unfortunately, this crucial difference between the two models is not reflected in the observable distribution function $e(x)$. Since the Dirac-sea contribution in the CQSM is nearly saturated by the δ-function singularity, it happens that the distributions $e(x)$ at $x\neq 0$ predicted by the two models are not extremely different from each other. This is the reason why the naive MIT bag model, which fails to explain the magnitude of the πN sigma term, can reproduce the empirical distribution $e(x)$ extracted from the CLAS data at least qualitatively.

Still, we will show that there are some qualitative and observable differences between the predictions of the CQSM and MIT bag model. The key observation here is that, for the spin-independent chiral-odd twist-3 distribution functions, the MIT bag model predicts no flavor dependence. That is, within the framework of the naive MIT bag model, we have

$$e^u(x)=e^d(x), \quad e^{\bar{u}}(x)=e^{\bar{d}}(x), \tag{83}$$

or, more specifically,

$$e^u(x)+\frac{1}{4}e^{\bar{d}}(x)=e^d(x)+\frac{1}{4}e^{\bar{u}}(x). \tag{84}$$

Such equalities can be expected to hold only in the fictitious limit of $N_c\to\infty$. As is in fact the case with the CQSM, for a finite value of N_c, the isovector distribution $e^{(T=1)}(x)=e^u(x)-e^d(x)$ does not vanish, so that we definitely expect that

$$e^u(x)+\frac{1}{4}e^{\bar{d}}(x)\neq e^d(x)+\frac{1}{4}e^{\bar{u}}(x). \tag{85}$$

Figure 10 shows the comparison of the predictions of the two models for the distributions $e^u(x)+\frac{1}{4}e^{\bar{d}}(x)$ and $e^d(x)+\frac{1}{4}e^{\bar{u}}(x)$ evolved to the energy scale of CLAS experiment: i.e., $Q^2\simeq 1.5$ GeV^2 from the initial energy scale of the model $Q^2_{ini}\simeq 0.30$ GeV^2. The solid and dashed curves here stand for the predictions of the CQSM, respectively, for $e^u(x)+\frac{1}{4}e^{\bar{d}}(x)$ and $e^d(x)+\frac{1}{4}e^{\bar{u}}(x)$. On the other hand, the dot-dashed curve represents the prediction of the MIT bag model, which gives an identical answer for both these com-

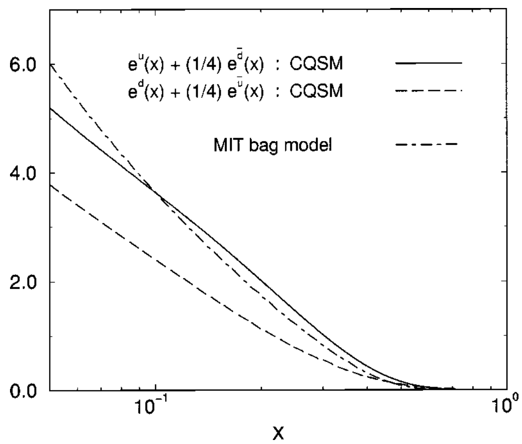

FIG. 10. The predictions of the CQSM for $e^u(x)+(1/4)e^{\bar{d}}(x)$ and $e^d(x)+(1/4)e^{\bar{u}}(x)$ evolved to the energy scale $Q^2 \simeq 1.5$ GeV^2 of the CLAS data from the initial scale of the model $Q^2_{ini} \simeq 0.30$ GeV^2. Also shown for comparison is the prediction of the MIT bag model evolved to the same scale from somewhat lower energy scale of $Q^2_{ini} \simeq 0.16$ GeV^2.

binations of distributions. One sees that the CQSM predicts a sizably large difference between the two distributions $e^u(x) + \frac{1}{4}e^{\bar{d}}(x)$ and $e^d(x) + \frac{1}{4}e^{\bar{u}}(x)$, in sharp contrast to the MIT bag model. In principle, the possible differences of these two distributions can be detected by performing a comparative analysis of the semi-inclusive $\pi^{\pm}$ and π^0 productions.

IV. SUMMARY AND CONCLUSION

In summary, we have given theoretical predictions for the chiral-odd twist-3 distribution function $e^a(x)$ of the nucleon with each flavor a on the basis of the chiral quark soliton model. A prominent feature of the isosinglet combination of the distributions, $e^u(x)+e^d(x)$, is that its first moment is proportional to the familiar πN sigma term and that it contains a delta-function singularity at $x=0$. In the previous study based on the derivative expansion technique, we demonstrated that the physical origin of this singularity can be traced back to the long-range quark-quark correlation of scalar type, which signals the spontaneous chiral symmetry breaking of the QCD vacuum. The present calculation, without recourse to the derivative-expansion-type approximation, has revealed the following facts. The isosinglet distribution $e^u(x)+e^d(x)$ consists of two parts: i.e., the contribution of N_c valence level quarks and that of the Dirac sea quarks in the hedgehog mean field. The former takes a familiar shape of distribution which has a peak around the value of $x \simeq 1/3$. On the other hand, the latter certainly contains a δ-function-type singularity at $x=0$, but it also has nontrivial support for $x \neq 0$. The isovector distribution $e^u(x)-e^d(x)$ also consists of the valence and Dirac-sea contributions. For this distribution, however, no delta-function-type singularity is observed, which means that it is a regular function in all the range of x.

The moment sum rules of $e(x)$ provide us with valuable information concerning the basic dynamical content of the model in view of the underlying theory: i.e., QCD. We showed that the first-moment sum rule for $e^u(x)+e^d(x)$ is satisfied within the model if and only if the delta-function singularity is properly taken into account. Note however that the delta-function term alone does not saturate the first moment or the πN sigma-term sum rule in contrast to the previous argument based on the framework of the perturbative QCD. We also pointed out that the second-moment sum rule for $e^u(x)+e^d(x)$ does not vanish even in the chiral limit in contrast to the QCD equation-of-motion argument. In our opinion, this violation of the second-moment sum rule does not necessarily show a defect of the model. It is rather to be interpreted as showing the limitation of the perturbative analysis as a tool of handling a bound-state problem and/or the problem of masses nonperturbatively generated by the mechanism of the spontaneous chiral symmetry breaking. We have also shown that the model prediction for the first moment of the isovector distribution $e^u(x)-e^d(x)$ comes out to be order of magnitude consistent with the phenomenological estimate obtained from the nonelectromagnetic neutron-proton mass difference.

It was shown that the theoretical predictions for the distribution $e^u(x)+\frac{1}{4}e^{\bar{d}}(x)$ are in a good agreement with the corresponding empirical information extracted from the CLAS data for the semi-inclusive π^+ production under the assumption of the Collins mechanism dominance. This agreement, combined with our analysis explained in the text, implies the existence of a δ-function singularity at $x=0$ in the isosinglet distribution $e^u(x)+e^d(x)$, although a definite conclusion must await for more complete measurements and a more thorough understanding of the reaction mechanism that generates the beam single-spin asymmetry in semi-inclusive pion production.

Finally, we compare our theoretical predictions with those of the MIT bag model. As shown in the body of the paper, the two models give accidentally close predictions for the distribution function $e^u(x)+\frac{1}{4}e^{\bar{d}}(x)$ at $x \neq 0$. We have shown, however, that the CQSM predicts a sizably large difference between the two distributions $e^u(x)+\frac{1}{4}e^{\bar{d}}(x)$ and $e^d(x)+\frac{1}{4}e^{\bar{u}}(x)$, for which the MIT bag model makes no difference. The predicted sizable difference between the two combinations of distributions will be detected by performing a comparative experimental analysis of semi-inclusive $\pi^{\pm}$ and π^0 production.

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